2024 AGMC Tianyi Summer Cup Math Competition

【Individual Round - Junior Group】

Question Sheet

(Full Marks: 100 points Duration: 80 minutes)

Read the following instructions carefully before you start the exam.

I. The competition consists of 25 questions with a total score of 100 points and a duration of 80 minutes.

II. Use Rainclassroom to save your responses. Answers written on the question sheet or draft paper will be considered invalid.

III. This competition is an online open-book exam, and consulting paper materials is allowed. The use of electronic devices such as computers, mobile phones, calculators, etc. is prohibited. Any use of electronic devices will result in disqualification.

IV. This competition is an individual round. Consequently, any collaboration is prohibited. Any forms of collaboration will result in disqualification.

V. The sequence of questions in Rainclassroom is randomized and does not correspond to the order presented in this PDF.

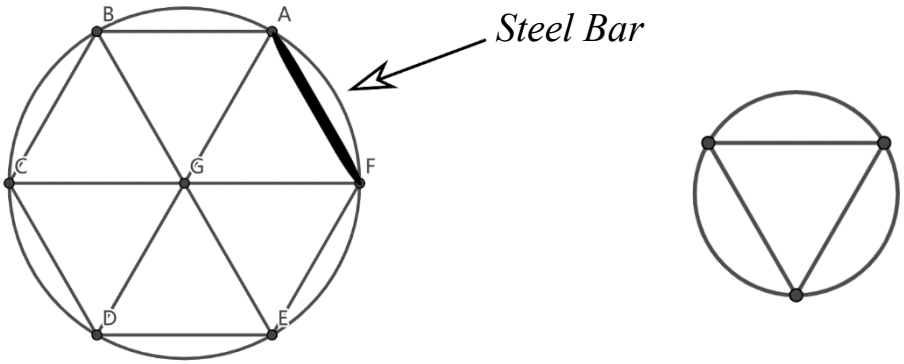
VI. Rainclassroom features an automated grading system, allowing participants to access their scores within five minutes after the conclusion of the competition.

Section I 【Easy Level】

(*This section contains 10 questions, each worth 3 points, for a total of 30 points*)

1. The picture on the left shows a large wheel with a cross-sectional area of 75π, consisting of 12 steel bars. Now a car mechanic will remove 3 of the steel bars and install them on the small wheel on the right. The cross-sectional area of the small wheel is ( ).

A. 10π B. 15π C. 20π D. 25π



2. The unit digit of the number $2023^{2024}+2024^{2023}$ is ( ).

A. 1 B. 3 C. 5 D. 7

3. An even function refers to a function whose graph is symmetric with respect to the y-axis. David believes that all quadratic functions are even functions. Given the function $y=ax^2+bx+c$. Which of the following statement is correct? ( )

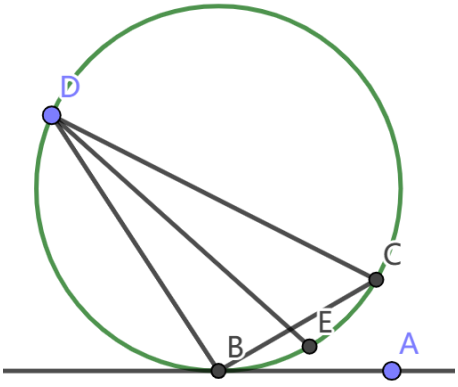
A. David's view is correct.

B. David's view is incorrect; when $a=0$, $b\neq 0$, the function is an even function.

C. David's view is incorrect; when $a\neq 0$, $b\neq 0$, the function is an even function.

D. David's view is incorrect; when $a\neq 0$, $b=0$, the function is an even function.

4. As shown in the figure, line AB is tangent to a circle at point B, $\angle ABC=30°$, point D is a point on the superior arc BDC, connected BD, CD, the angle bisector DE of $\angle BDC$ intersects the circle except D at point E, then $\angle BDE=$ ( ).



A. 10° B. 15°

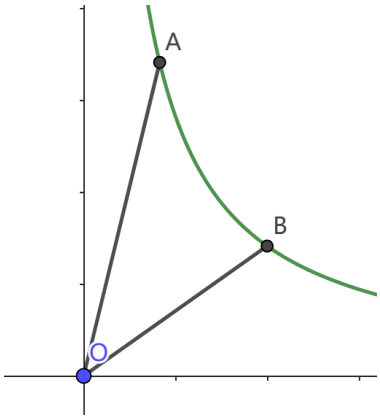
C. 20° D. 30°

5. The two real roots of the quadratic equation $x^2 + ax + 2 = 0$ $(a \neq 0)$ are x_1 and x_2. The value range of $\frac{1}{x_1} + \frac{1}{x_2}$ is ( ).

A. $-1 \leq \frac{1}{x_1} + \frac{1}{x_2} \leq 1$ B. $\frac{1}{x_1} + \frac{1}{x_2} \leq -1$ or $\frac{1}{x_1} + \frac{1}{x_2} \geq 1$

C. $-\sqrt{2} \leq \frac{1}{x_1} + \frac{1}{x_2} \leq \sqrt{2}$ D. $\frac{1}{x_1} + \frac{1}{x_2} \leq -\sqrt{2}$ or $\frac{1}{x_1} + \frac{1}{x_2} \geq \sqrt{2}$

6. The figure is the image of the function $y = \frac{k}{x}(k > 0,\ x > 0)$, points A and B are two points on the image, x_A is the horizontal coordinate of point A, x_B is the horizontal coordinate of point B, $x_A < x_B$. Connect OA, OB, if $S_{\triangle AOB} = k$, $\frac{x_A}{x_B} = ($).

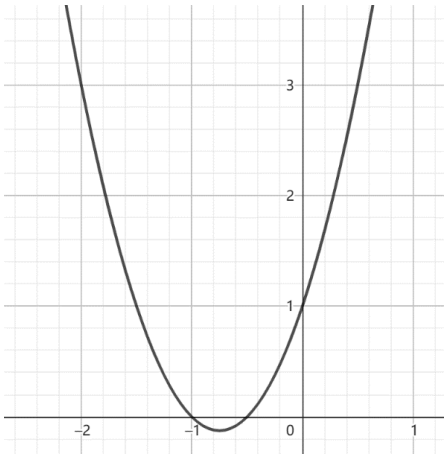


A. $\sqrt{2} - 1$ B. $\frac{1}{2}$ C. $2 - \sqrt{2}$ D. $\frac{\sqrt{2}}{2}$

7. The cake shop owner sells cupcakes and finds that when the price of a cupcake is 10$, the daily sales volume is 200. Thereafter, for every 1$ increase in price, the daily sales volume decreases by 10. Therefore, she decides to increase the price of the cupcakes to earn more money. The maximum amount the cake shop owner can earn per day by selling cupcakes is ( ).

A. 1500$ B. 2000$ C. 2250$ D. 2500$

8. The figure shown is a partial image of a quadratic function $y = ax^2 + bx + c$, then the analytic expression of this quadratic function is ( ).



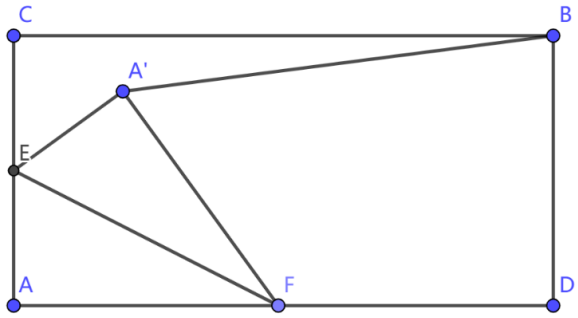
A. $y = 2x^2 + 3x + 1$ B. $y = 3x^2 + 2x + 1$

C. $y = x^2 + 2x + 1$ D. $y = x^2 + 3x + 1$

9. The range of real number a that makes the equation $\frac{a}{x}=\frac{x}{x\text{-}1}$ have no solution in the real number domain is ( ).

A. $0<a<2$ B. $0\le a<2$ C. $0<a<4$ D. $0\le a<4$

10. As shown in the figure, the quadrilateral $ACBD$ is a rectangle, where AC=2, AD=4, E is the midpoint of AC, and F is a moving point on AD. After folding $\triangle AEF$ in half along line EF, we get $\triangle A'EF$. Connect $A'B$, then $A'B_{\min}$= ( ).



A. $\sqrt{17}-1$ B. $\sqrt{17}-2$ C. $2\sqrt{2}$ D. $2\sqrt{2}-1$

Section II 【Medium Level】

(*This section contains 8 questions, each worth 4 points, for a total of 32 points*)

11. The vertices of a positive n-sided shape arranged counterclockwise are $P_1,P_2,P_3\ldots P_n$, $\angle P_1P_5P_6=175°$, then n =( ).

A. 18 B. 36 C. 180 D. 216

12. As shown in the figure, in $\triangle ABC$, $\angle A$=3$\angle C$, AB=4, BC=5, then AC= ( ).

A. $\sqrt{2}$ B. $\frac{3}{2}$ C. $\sqrt{3}$ D. 2

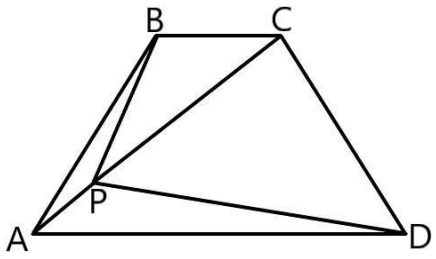
13. Let a, b, c be nonzero constants. There are three quadratic equations：$ax^2+bx+c=0$，$bx^2+cx+a=0$，$cx^2+ax+b=0$，the maximum sum of the number of real roots of the three equations is ( ).

A. 2 B. 3 C. 4 D. 6

14. Prushka conducted a survey on the number of pens she and 5 other classmates own. Prushka has x pens, while the other five classmates have 1, 2, 5, 5, and 7 pens, respectively. Therefore, the data set is 1, 2, 5, 5, 7, x. If the average of this data set is exactly equal to the number of pens one of the classmates owns, the sum of all possible values of x is ( ).

A. 32 B. 36 C. 40 D. 46

15. As shown in the figure, quadrilateral $ABCD$ is an isosceles trapezoid, $AD \parallel BC$, $AB=CD$, and P is a point within the quadrilateral $ABCD$. Connect AP, BP, CP, DP. If $AP=1$, $BP=2$, $CP=3$, $DP=4$, which of the following statements is correct? ( )

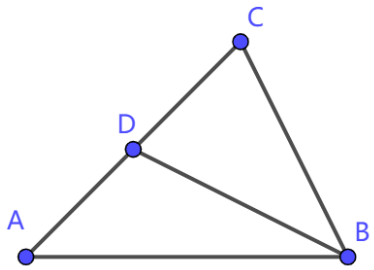


A. $AD=3BC$ B. $AB=2BC$ C. $1 \le BC \le 2$ D. $1 \le CD \le 7$

16. In $\triangle ABC$, $\angle A=45°$, $AC=2$. Let D be the midpoint of AC, and connect BD. The minimum value of $BC+BD$ is ( ).

A. 2 B. $\sqrt{5}$

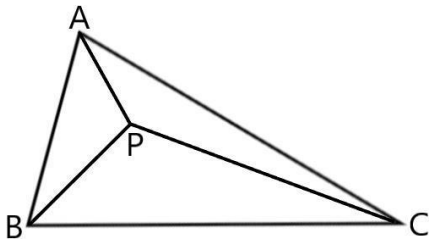
C. $\sqrt{6}$ D. $\sqrt{7}$



17. In a 3×3 grid, select 3 grids for coloring, requiring that there are no adjacent two colored grids. Two adjacent grids refer to two grids sharing a side. There are ( ) ways to select grids for coloring.

A. 16 B. 18 C. 20 D. 22

18. As shown in the figure, in $\triangle ABC$, $AC=BC=1$, $\angle ACB=30°$, and P is a moving point inside $\triangle ABC$. Connect AP, BP, CP, the minimum value of $3AP+4BP+5CP$ is ( ).



A. $\sqrt{37}$ B. $\sqrt{43}$ C. $4\sqrt{3}$ D. $5\sqrt{2}$

Section III 【Difficult Level】

(*This section contains 4 questions, each worth 5 points, for a total of 20 points*)

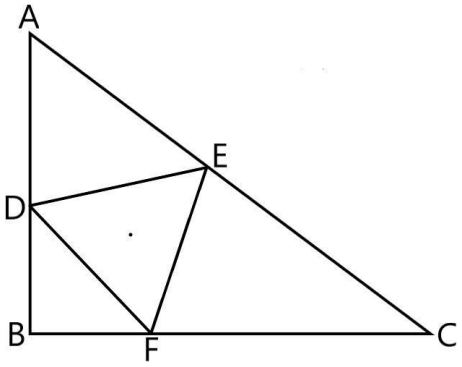
19. Let n be a positive integer not greater than 1000 and satisfy that the least common multiple of n and 9 is a perfect square. The number of n is ( ).

A. 31 B. 43 C. 55 D. 67

20. The indeterminate equation $x^2+xy+y^2=x^2y^2$ has ( ) pairs of integer solutions.

A. 1 B. 2 C. 3 D. 4

21. As shown in the figure, it is known that $\triangle ABC$ is a right triangle, in which $\angle ABC=90°$, $AB=3$, $BC=4$. Points D, E, F are the three points on the sides AB, AC, BC, respectively. Connect DE, DF, EF. If the incenter of $\triangle ABC$ coincides with the centroid of $\triangle DEF$, $S_{\triangle DEF\min}=$ ( ).



A. $\frac{11}{8}$ B. $\frac{10}{7}$ C. $\frac{\sqrt{7}}{2}$ D. $\frac{4\sqrt{2}}{3}$

22. There is are ( ) methods to divide the natural numbers 1-14 into 7 groups, and satisfy that each group contains 2 numbers, and one of the 2 numbers is at least twice the other number.

A. 108 B. 120 C. 132 D. 144

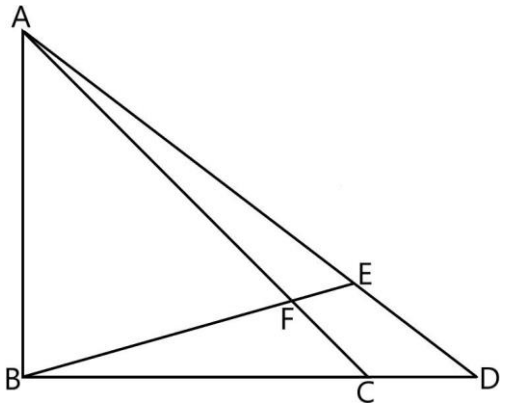
Section IV 【Challenging Level】

(*This section contains 3 questions, each worth 6 points, for a total of 18 points*)

23. Let x, y be real numbers，and $\frac{x^2+y^2}{x+y}=4$，$\frac{x^4+y^4}{x^3+y^3}=2$，then $\frac{x^6+y^6}{x^5+y^5}=$（ ）.

A. $5\pm\sqrt{23}$ B. $8\pm2\sqrt{21}$ C. $10\pm2\sqrt{17}$ D. $12\pm2\sqrt{13}$

24. As shown in the figure, $\triangle ABC$ is an isosceles right triangle, where $AB=BC=2$, $\angle ABC=90°$. Point D is a point on the extended line of BC, connected to AD, point E is a point on segment AD. BE and AC intersect at point F, $AD=2BF$, $\tan\angle AEB=\frac{4}{3}$, then CF= ( ).



A. $\frac{2\sqrt{2}}{5}$ B. $\frac{\sqrt{3}}{3}$ C. $\frac{3}{5}$ D. $\frac{3\sqrt{2}}{7}$

25. Let a, b, c, d be different positive integers, where $a+b=c+d$. The least common multiple of a, b, c, d is less than 1000, the maximum value of $a+b$ is ( ).

A. 534 B. 581 C. 617 D. 664